

Dependency atlas

A visual atlas generated from the companion JSON, with exact source locations and external-input status.

158 nodes / 278 curated logical edges / 220 indexed references. The 21 case contracts use stable S/N IDs. All numbered statements retain their original labels or receive result:<number> IDs.

01 / Original

original/paper.pdf
Complete human-readable narrative.
original/paper.tex: frozen source snapshot.

02 / AI edition

ai/paper_ai.pdf
Four takeaway pages, then full proof text.
Original theorem numbers preserved.

03 / JSON

theorem_graph/theorems.json
LaTeX statements, source lines, PDF pages,
logical dependencies and external status.

How to read the graph

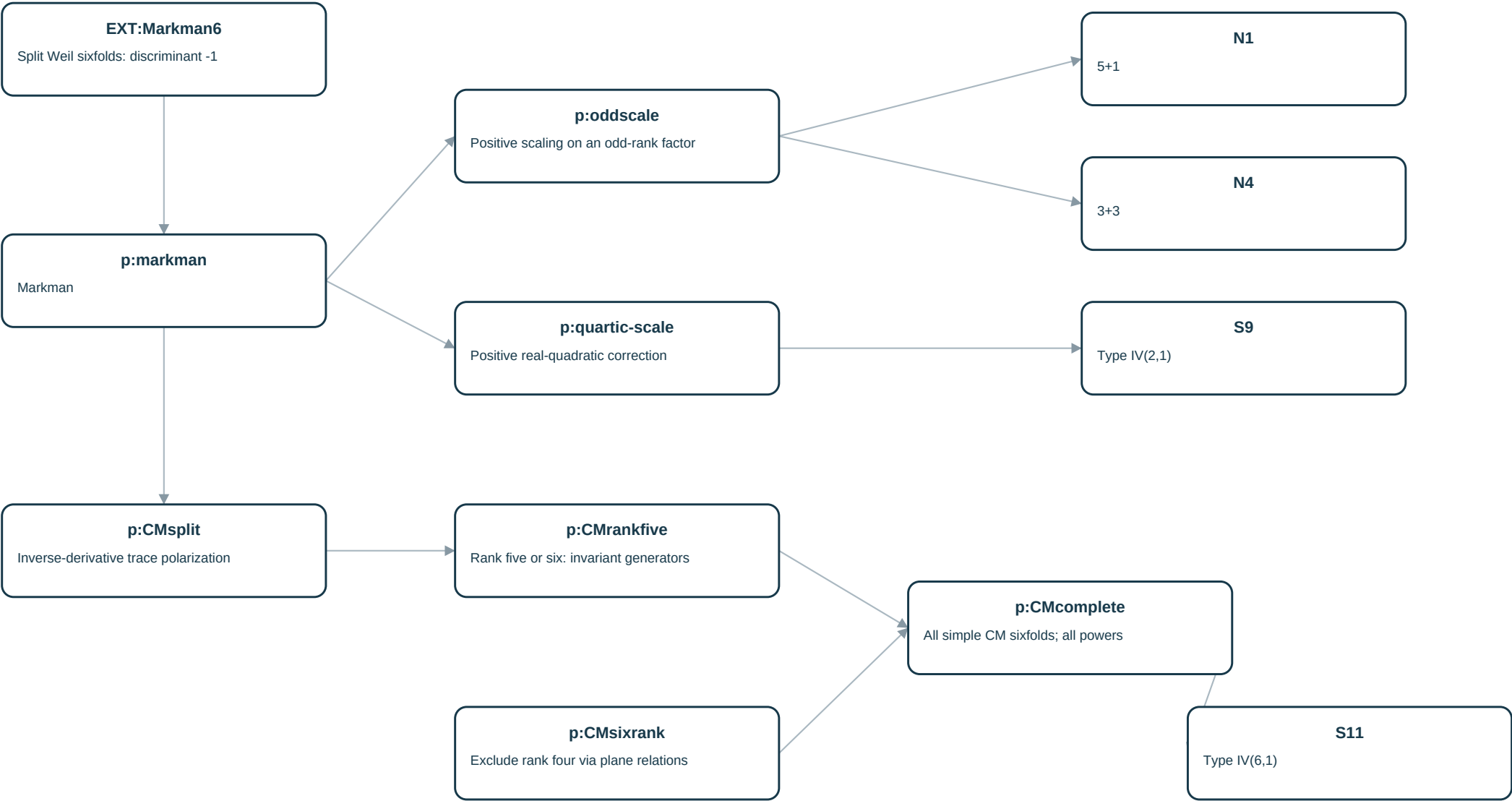
Arrows run from a prerequisite to a dependent. Each panel is a selected subgraph of the same JSON; omitted arrows are not evidence of independence. Red borders mark withdrawn external assertions. Ochre borders mark conclusions whose curated route uses a conditional input. Blue marks the other indexed routes, including cited geometric theorems that are not reproved here.

Two different edge sets

Curated logical edges describe sufficient central proof routes. Reference edges only record explicit LaTeX references; they can be forward pointers, examples or alternative routes. They are not automatically promoted to proof dependencies. Uncurated results are explicitly marked `references_indexed_only`.

Polarization and CM routes

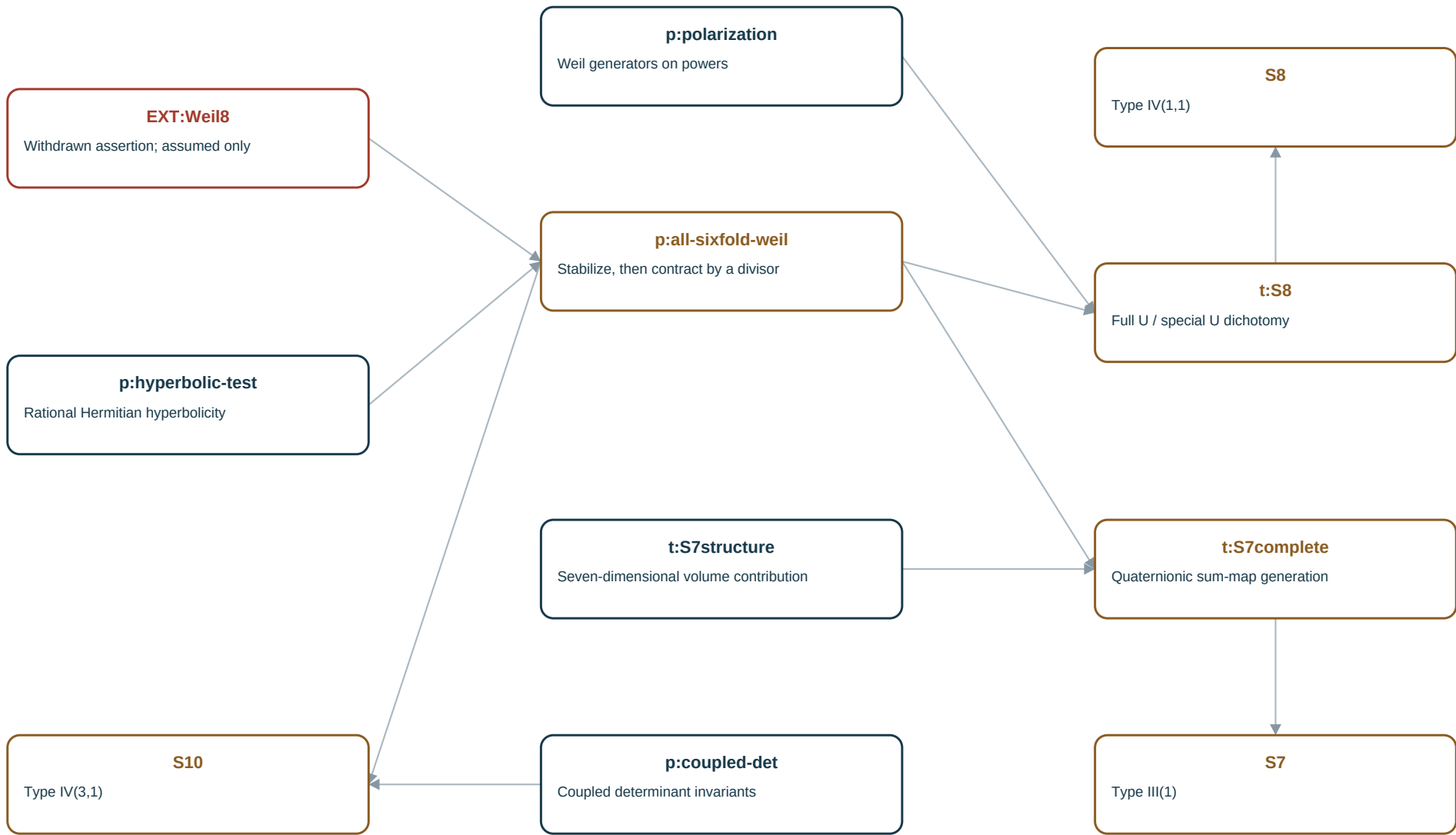
The key S9 and S11 constructions use the cited split sixfold theorem, not the withdrawn eightfold/KS/Aux assertions.



N4 also uses the actual Hodge-circle test; S9 also uses determinant invariants. Only their polarization branch is shown. The S11 rank bound does not itself prove cycle algebraicity.

Stabilization and unitary generators

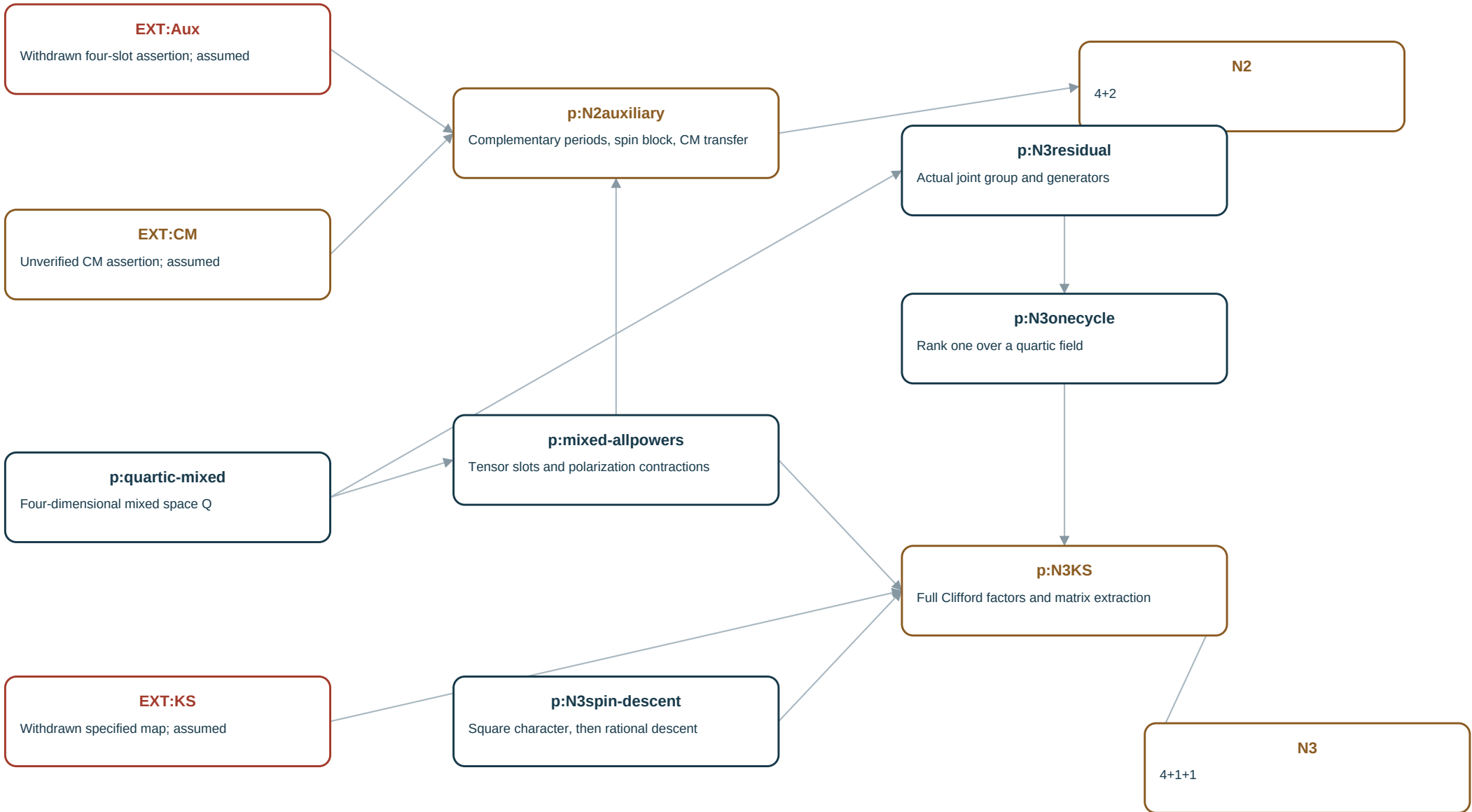
The eightfold assertion is used only as a conditional geometric input. The contraction back to the sixfold is an explicit algebraic operation.



The general-linear branches need no Weil algebraicity. Split subfamilies can use Markman directly. The complete S7, S8 and S10 routes shown in the case contracts use EXT:Weil8.

Residual mixed-cycle routes

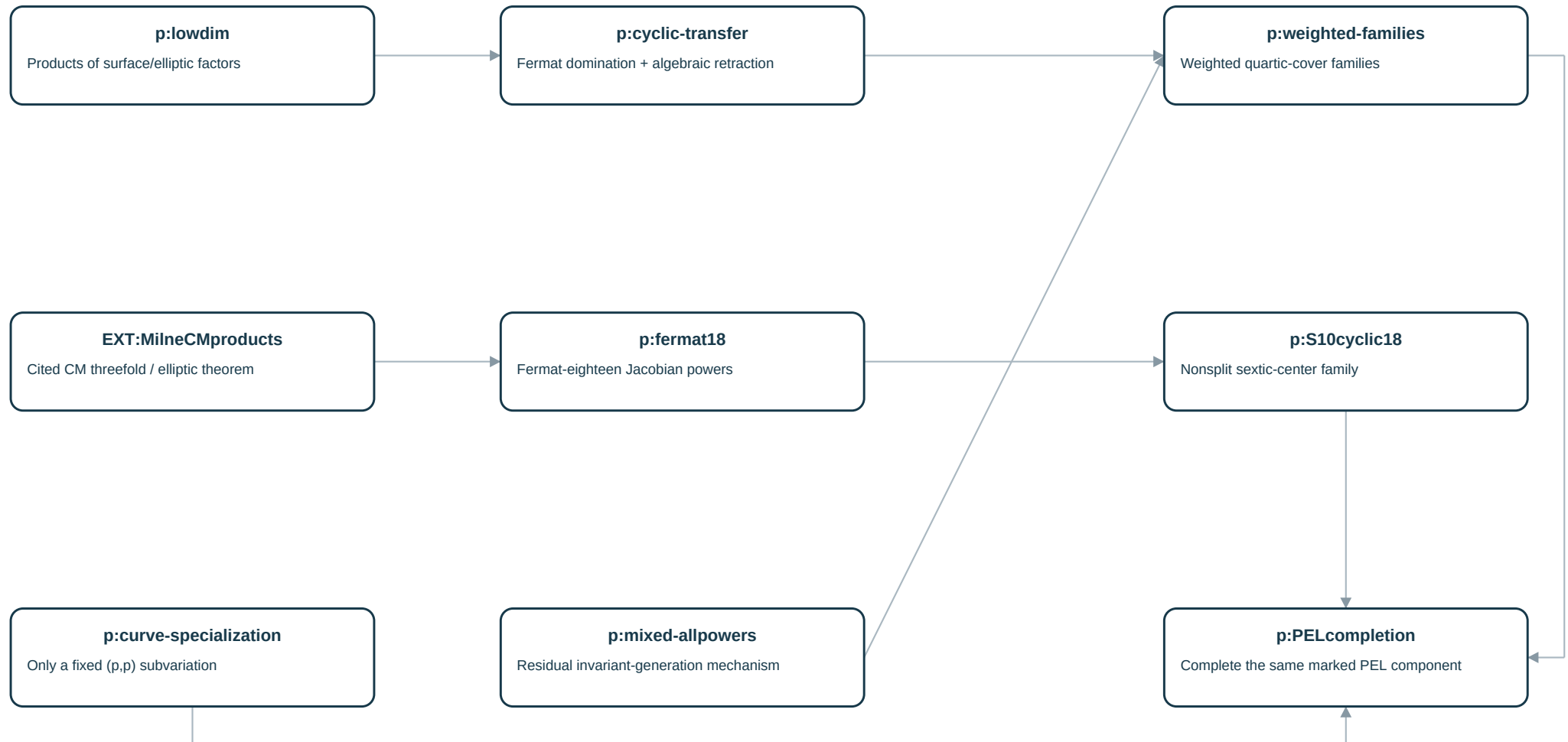
N2 and N3 have a common invariant model, but their geometric inputs and constructions differ. A CM transfer is not a consequence of KS alone.



The full N2/N3 case contracts also include independent, repeated-factor and CM strata. Their full-row conclusion is sixfold algebraicity; the displayed residual models additionally have all-powers results.

Cover geometry and specialization

These routes treat the represented geometric families and marked one-dimensional PEL components; they do not establish arbitrary rational Hermitian types.



For specialization, only classes in the fixed subvariation are transported. New Hodge classes appearing solely at special fibers are not covered by that statement. Not all classical prerequisites are drawn.

The 21 case contracts

Rows 1-11. A star marks a whole-row route using a new conditional input; it does not mark every stratum.

S1	Type I(1)	Full symplectic image; exclude the 2 x 6 tensor representation using the Hodge cocharacter and rational normal factors.	all powers
S2	Type I(2)	Small symplectic constituents and commutant/Goursat separation give the full product; apply pairing invariant theory.	all powers
S3	Type I(3)	The same full-product argument, with degree-three real multiplication.	all powers
S4	Type I(6)	Rank-two symplectic blocks, separated by the commutant, give divisor generators.	all powers
S5	Type II(1)	Quaternionic multiplicity space has full Sp_6 image; pairings generate every power.	all powers
S6	Type II(3)	Three rank-two quaternionic multiplicity blocks give the full symplectic product.	all powers
S7 *	Type III(1)	Find an imaginary-quadratic Weil plane, then use quaternionic endomorphisms and sum maps to generate exceptional volume tensors.	all powers
S8 *	Type IV(1,1)	Unbalanced signatures give a full unitary group; balanced signature leaves a Weil plane, then polarization of generators gives powers.	all powers
S9	Type IV(2,1)	Determine the determinant torus; correct the rational discriminant by a positive real-quadratic endomorphism.	all powers
S10 *	Type IV(3,1)	Determine the determinant torus; its exceptional Weil plane is handled by stabilization and contraction.	all powers
S11	Type IV(6,1)	Exclude Hodge rank four by a two-dimensional relation space; split the rank-five Weil form using inverse derivatives and interpolation.	all powers

Conditional input ledger: S7/S8/S10 -> Weil8; N2 -> Aux + CM; N3 -> KS + CM. Sixfold-only labels are conservative records of the stated full-row scope, not counterexamples to all-powers extensions.

The 21 case contracts

Rows 12-21. A star marks a whole-row route using a new conditional input; it does not mark every stratum.

N1	5+1	A prime-dimensional factor leaves at most one mixed Weil plane; odd-rank polarization adjustment makes it split.	sixfold as stated
N2 *	4+2	Separate independent strata. The residual quartic mixed model reduces to one cycle; Aux constructs it and a CM correspondence transfers it.	sixfold + residual powers
N3 *	4+1+1	Treat independent and repeated elliptic factors first; use a K3 realization, full Clifford matrix extraction and contraction in the residual model.	sixfold + residual powers
N4	3+3	Test the actual Hodge cocharacters of common quotients; the remaining determinant relation gives an odd-rank split Weil plane.	sixfold as stated
N5	3+2+1	Separate the surface, then use the threefold-elliptic fourfold Weil plane.	sixfold as stated
N6	3+1+1+1	Group elliptic isogeny classes; sum-map pullbacks of the fourfold Weil plane generate the possible mixed classes.	sixfold as stated
N7	2+2+2	Apply the product theorem for abelian varieties of dimension at most two.	all powers
N8	2+2+1+1	Apply the same low-dimensional product theorem, including repeated and CM factors.	all powers
N9	2+1+1+1+1	Apply the same low-dimensional product theorem.	all powers
N10	1+1+1+1+1+1	Collect elliptic isogeny classes; symplectic and opposite-character pairings generate the invariants.	all powers

Conditional input ledger: S7/S8/S10 -> Weil8; N2 -> Aux + CM; N3 -> KS + CM. Sixfold-only labels are conservative records of the stated full-row scope, not counterexamples to all-powers extensions.

Complete statement index

Original result numbers and stable JSON IDs / entries 1-23 of 129. O = original PDF page; A = AI PDF page.

P.1 lemma	p:isogeny	Prelemma P.1	O 4 / A 6
P.2 lemma	p:fft	Prelemma P.2	O 4 / A 7
P.3 lemma	p:small	Prelemma P.3	O 5 / A 7
P.4 lemma	p:goursat	Prelemma P.4	O 5 / A 7
P.5 proposition	p:realconstruction	Preproposition P.5	O 6 / A 7
P.6 proposition	p:quaternionconstruction	Preproposition P.6	O 6 / A 8
P.7 proposition	p:products	Preproposition P.7	O 6 / A 8
P.8 theorem	p:markman	Markman	O 7 / A 8
P.9 theorem	p:openai-inputs	New external inputs	O 7 / A 8
P.10 lemma	p:hyperbolic-test	Rational hyperbolicity	O 7 / A 8
P.11 theorem	p:all-sixfold-weil	All sixfold Weil planes from the eightfold input	O 7 / A 9
P.12 lemma	p:CMproducts-new	Products of CM factors	O 8 / A 9
P.13 lemma	p:polarization	Prelemma P.13	O 8 / A 9
P.14 lemma	p:HLsix	Prelemma P.14	O 9 / A 9
P.15 theorem	p:lowdim	Low-dimensional inputs	O 9 / A 9
P.16 lemma	p:oddscale	Odd-rank polarization adjustment	O 9 / A 10
P.17 lemma	p:quartic-scale	Real-quadratic polarization adjustment	O 10 / A 10
P.18 lemma	p:CMsplit	A split trace polarization for CM Weil sixfolds	O 10 / A 10
P.19 lemma	p:prime-ell	A prime-dimensional factor and an elliptic curve	O 11 / A 11
P.20 lemma	p:three-ell	A threefold and repeated elliptic factors	O 12 / A 11
P.21 lemma	p:surface-three	A surface separates from a threefold and elliptic curves	O 12 / A 11
P.22 lemma	p:smallunitary	Small unitary constituents	O 13 / A 12
1.1 theorem	t:S1	Theorem 1.1	O 14 / A 12

JSON preserves the full LaTeX statement, including assumptions, and one-based statement/proof source lines. references_indexed_only means no claim of a complete audited logical dependency list for that individual statement.

Complete statement index

Original result numbers and stable JSON IDs / entries 24-46 of 129. O = original PDF page; A = AI PDF page.

1.2 example	result:1.2	Example 1.2	O 14 / A 13
2.1 theorem	result:2.1	Theorem 2.1	O 14 / A 13
2.2 example	result:2.2	Example 2.2	O 15 / A 13
3.1 theorem	result:3.1	Theorem 3.1	O 15 / A 13
3.2 example	result:3.2	Example 3.2	O 15 / A 13
4.1 theorem	result:4.1	Theorem 4.1	O 16 / A 14
4.2 example	result:4.2	Example 4.2	O 16 / A 14
5.1 theorem	result:5.1	Theorem 5.1	O 16 / A 14
5.2 example	result:5.2	Example 5.2	O 16 / A 14
6.1 theorem	result:6.1	Theorem 6.1	O 17 / A 14
6.2 example	result:6.2	Example 6.2	O 17 / A 15
7.1 theorem	t:S7structure	Theorem 7.1	O 17 / A 15
7.2 theorem	t:S7complete	Complete Type III row	O 18 / A 15
7.3 theorem	t:S7split	The earlier split-subfield case	O 18 / A 15
7.4 corollary	p:S7onecycle	A single exceptional cycle in Type III	O 18 / A 15
7.5 example	ex:hamilton	Example 7.5	O 19 / A 16
7.6 remark	result:7.6	Remark 7.6	O 19 / A 16
7.7 proposition	p:quaternion-split-test	An arithmetic split-subfield criterion	O 19 / A 16
7.8 example	ex:quaternion-nonsplit	A quaternionic family outside every split subfield	O 20 / A 16
8.1 theorem	t:S8	Theorem 8.1	O 20 / A 17
8.2 example	result:8.2	Example 8.2	O 21 / A 17
9.1 theorem	result:9.1	Theorem 9.1	O 21 / A 17
9.2 example	result:9.2	Example 9.2	O 22 / A 18

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Complete statement index

Original result numbers and stable JSON IDs / entries 47-69 of 129. O = original PDF page; A = AI PDF page.

9.3 proposition	p:coupled-det	Proposition 9.3	O 22 / A 18
9.4 example	result:9.4	A split coupled quartic-CM family	O 23 / A 18
9.5 example	result:9.5	A nonsplit polarization that can be corrected	O 23 / A 18
10.1 theorem	result:10.1	Theorem 10.1	O 24 / A 19
10.2 example	result:10.2	Example 10.2	O 24 / A 19
10.3 example	result:10.3	A split coupled sextic-CM family	O 25 / A 19
10.4 lemma	p:fermat18	All powers of the Fermat-eighteen Jacobian	O 25 / A 20
10.5 theorem	p:S10cyclic18	A nonsplit cyclic-eighteen family	O 25 / A 20
11.1 proposition	t:CM	Proposition 11.1	O 28 / A 21
11.2 proposition	p:CMtest	An exact finite test for every specified CM type	O 28 / A 21
11.3 remark	result:11.3	Remark 11.3	O 29 / A 22
11.4 corollary	p:CMquartic	A checkable degenerate CM stratum	O 29 / A 22
11.5 corollary	p:CMrankfive	All CM sixfolds of Hodge rank at least five	O 29 / A 22
11.6 lemma	p:CMSixrank	A rank bound without a Galois hypothesis	O 29 / A 22
11.7 theorem	p:CMcomplete	All simple CM sixfolds	O 30 / A 23
11.8 remark	result:11.8	Independent finite cross-check	O 30 / A 23
11.9 corollary	p:CMabelian	All abelian Galois CM fields of degree twelve	O 31 / A 23
11.10 corollary	p:CMgalois	All Galois CM fields of degree twelve	O 31 / A 23
11.11 example	result:11.11	Exact matrix certificate for the trace construction	O 32 / A 24
11.12 example	ex:CM13	Example 11.12	O 33 / A 24
11.13 remark	result:11.13	Remark 11.13	O 33 / A 25
11.14 example	result:11.14	A degenerate CM sixfold with algebraic exceptional classes	O 33 / A 25
12.1 theorem	result:12.1	Theorem 12.1	O 34 / A 25

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Complete statement index

Original result numbers and stable JSON IDs / entries 70-92 of 129. O = original PDF page; A = AI PDF page.

12.2 example	result:12.2	Example 12.2	O 35 / A 25
12.3 example	result:12.3	A genuinely exceptional $\$5+1\$$ product	O 35 / A 26
13.1 corollary	p:N2CM-new	All CM--CM products in N2	O 35 / A 26
13.2 example	ex:N2CM-new	A CM fourfold times a CM surface	O 35 / A 26
13.3 proposition	result:13.3	Proposition 13.3	O 36 / A 26
13.4 proposition	p:N2strata	Two further complete strata	O 36 / A 26
13.5 proposition	p:N2galois	A splitting-field separation test	O 36 / A 26
13.6 proposition	p:N2nonCMstructure	All residual non-CM-fourfold groups in N2	O 36 / A 27
13.7 proposition	p:N2CMrankthree	Rank-three CM fourfolds are independent of surfaces	O 37 / A 27
13.8 proposition	p:N2CMstructure	All CM-fourfold couplings in N2	O 38 / A 27
13.9 remark	result:13.9	A finite-kernel obstruction to an automatic reduction	O 39 / A 28
13.10 lemma	p:quartic-mixed	The quartic mixed invariant calculation	O 40 / A 28
13.11 proposition	p:mixed-allpowers	Tensor contraction for every residual mixed model	O 40 / A 29
13.12 lemma	p:cyclic-transfer	A Fermat correspondence for weighted cyclic covers	O 41 / A 29
13.13 theorem	p:N2cyclic	A complete cyclic-five-cover family in N2	O 43 / A 31
13.14 theorem	p:weighted-families	All weighted quartic-cover families	O 44 / A 31
13.15 lemma	p:curve-specialization	Specializing a fixed Hodge subvariation over a curve	O 44 / A 31
13.16 theorem	p:PELcompletion	Completion of the represented one-dimensional PEL types	O 45 / A 32
13.17 example	ex:weighted-newtypes	Two additional marked rational Hermitian types	O 46 / A 32
13.18 remark	result:13.18	Finite branch catalogue and its precise scope	O 46 / A 32
13.19 theorem	p:N2auxiliary	Residual N2 from the auxiliary-spin tensor	O 47 / A 33
13.20 example	result:13.20	Example 13.20	O 49 / A 34
14.1 corollary	p:N3CM-new	All remaining CM-fourfold cases in N3	O 49 / A 34

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Complete statement index

Original result numbers and stable JSON IDs / entries 93-115 of 129. O = original PDF page; A = AI PDF page.

14.2 example	result:14.2	Distinct elliptic fields with a CM fourfold	O 49 / A 35
14.3 proposition	result:14.3	Proposition 14.3	O 50 / A 35
14.4 proposition	p:N3strata	Quadratic-center and independent strata	O 50 / A 35
14.5 proposition	p:N3repeated	All repeated elliptic factors	O 50 / A 35
14.6 proposition	p:N3residual	The full residual structure in N3	O 51 / A 36
14.7 corollary	p:N3onecycle	A single-cycle criterion for all residual N3 cases	O 52 / A 36
14.8 theorem	p:N3cyclic	Two coupled cyclic-cover families in N3	O 52 / A 36
14.9 corollary	p:cyclic-allparameters	All parameters in the three cyclic-cover families	O 53 / A 37
14.10 lemma	p:N3spin-descent	Connected Spin descent	O 54 / A 37
14.11 theorem	p:N3KS	Residual N3 under the specified KS correspondence	O 54 / A 37
14.12 remark	result:14.12	Logical scope of Theorem~\ref{p:N3KS}	O 56 / A 39
14.13 example	result:14.13	Example 14.13	O 56 / A 39
14.14 example	result:14.14	A mixed Weil plane requiring both elliptic factors	O 56 / A 39
15.1 proposition	result:15.1	Proposition 15.1	O 57 / A 39
15.2 lemma	p:CMthreepair	Two sextic CM factors: the actual Hodge-circle test	O 57 / A 39
15.3 proposition	p:N4strata	Threefold strata	O 58 / A 40
15.4 theorem	p:N4complete	The complete $\$3+3\$$ row	O 59 / A 40
15.5 example	result:15.5	Example 15.5	O 60 / A 41
15.6 example	result:15.6	Two nonisogenous unitary threefolds	O 60 / A 41
15.7 example	result:15.7	Two CM threefolds with a mixed Weil plane	O 60 / A 41
16.1 theorem	result:16.1	Theorem 16.1	O 61 / A 41
16.2 example	result:16.2	Example 16.2	O 61 / A 42
16.3 example	ex:N5weil	A fourfold Weil plane inside the $\$3+2+1\$$ row	O 61 / A 42

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Complete statement index

Original result numbers and stable JSON IDs / entries 116-129 of 129. O = original PDF page; A = AI PDF page.

17.1 theorem	result:17.1	Theorem 17.1	O 61 / A 42
17.2 example	result:17.2	Example 17.2	O 61 / A 42
17.3 example	result:17.3	Repeated matching elliptic factors	O 62 / A 42
18.1 theorem	result:18.1	Theorem 18.1	O 62 / A 42
18.2 example	ex:CMSurface	An explicit repeated CM surface	O 62 / A 42
19.1 theorem	result:19.1	Theorem 19.1	O 63 / A 43
19.2 example	result:19.2	A surface square and an elliptic square	O 63 / A 43
20.1 theorem	result:20.1	Theorem 20.1	O 63 / A 43
20.2 example	result:20.2	One CM surface and four equal elliptic factors	O 63 / A 43
21.1 theorem	result:21.1	Theorem 21.1	O 63 / A 43
21.2 example	result:21.2	Example 21.2	O 64 / A 44
P.23 theorem	p:conditional-sixfold	Conditional sixfold conclusion	O 64 / A 44
Tool 1 proof_tool	TOOL:determinant-projector	General quartic determinant projector	O 43 / A 30
Tool 2 proof_tool	TOOL:quartic-signature-group	General quartic signature group dichotomy	O 42 / A 30

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